Review Article

THE RELATIONSHIP BETWEEN NUMBER SENSE AND METACOGNITION

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ABSTRACT

It is increasingly recognized that higher order thinking skills are necessary to meet the challenges in the 21st century. Because in order to reach information, individuals need to have skills such as problem solving, organizing their own learning, transferring the acquired knowledge to new situations, reasoning, critical thinking, developing creative and original strategies, flexible thinking, estimation and mental computation. In recent years, these skills are expressed by number sense and metacognition. In literature review, there are several research findings that there is a significant relationship between mathematics achievement with number sense and metacognition. These findings suggest a relationship between these two concepts. Starting from this point the purpose of this study is to explain the concepts of metacognition and number sense and to examine the relationship between these two concepts. This study was designed in the form of a literature review based on synthesizing the data. As a result of the theoretical reviews, it was concluded that number sense and metacognition are supports both mathematics achievement and the mathematical thinking. Number sense and metacognition are skills based on flexible thinking.

Keywords: Mathematics, number sense, metacognition.

INTRODUCTION

In today's developing conditions, so many changes have been experienced in many fields such as the importance of information, ways of accessing information, learning-teaching approaches and technology. In particular, the developments in science and technology make it possible to reach knowledge through individual efforts rather than passively acquiring it. In order to reach the information, individuals need to have skills like problem solving, organizing their own learning, transferring the acquired knowledge to new situations, reconsidering, reasoning, critical thinking, developing creative and original strategies and also they should be able to use these skills in their social lives. Mathematics has a great importance in the acquisition of these skills and in the use of them in daily life situations.

Approaching around specific objectives to the mathematics that we will need life long occurs in school years. The skills required to be acquired by individuals through school mathematics are carried out within the curriculum. Through the mathematics curriculum that was updated according to the requirements of the era and applied in 2005, it has been aimed to be acquired mathematical skills such as associating concepts, reasoning, thinking flexibly, problem solving, estimation, and mental computation. The concept of Number Sense, which is not clearly expressed but is given to students by means of components, is included in the program (MEB, 2005). It has been continued to update the program during the process and the last update was made in 2018. In order to enable students to use metacognitive skills in the mathematics curriculum, which was updated in 2018, it was associated between the subjects and the pre-learning and integrated into other disciplines and daily life in the context of values, skills and competences; in this way, more importance was given to the development of metacognitive knowledge and skills than the previous ones (MEB, 2018).

Metacognition which is expressed as a higher-order thinking (Woolfolk, 2005; akt: Kacar & Sarıçam, 2015); with the Number Sense which we encounter as important concepts in school mathematics and which is centered on mathematics education and which is emphasized as the main objective of school mathematics and which will have a key content in the field of mathematics education in 21st century (Yang ve Li, 2008; NCTM, 2000; akt: Yang, 2003); have taken place in mathematics curriculum.

Considering that there is a significant relationship between maths achievement and number sense (Çekirdekci, Şengül & Doğan, 2016; Harç, 2010; Jordan, Glutting ve Ramineni, 2009), and metacognition (Aşık & Sevimli, 2015; Memiş & Arıcan, 2013), it is thought that it is necessary to investigate the relationship between these two concepts.

The aim of this study is to explain the concepts of metacognition and number sense which are frequently emphasized in recent years in mathematics education, to explain their importance in mathematics education and to examine the relationship between number sense and metacognition.

METHOD

In this study, the relationship between number sense and metacognition is explained within the context of the literature. Therefore this study was designed in the form of a literature review based on synthesizing the data. In this study which has a theoretical quality current situation descriptions are analyzed within the scope of related literature for this reason, in this study, document analysis which is one of the qualitative data collection methods has been applied. Document analysis covers the analysis of written materials with information on the objectives of the research (Yıldırım ve Şimsek, 2011). In this study, the related literature was searched and obtained resaearches are examined and the relationship between number sense and metacognition was discussed.

CONCEPTUAL FRAMEWORK

Number Sense

The number sense of which first used date as a term in mathematics literature is unknown was firstly mentioned in the book named 'Curriculum and Evaluation Standarts For School Mathematics 'by National Council of Teachers of Mathematic'. The number sense in the book is expressed as intuition about number relationships that help children to evaluate the logical appropriateness of mathematical computational results and support the solutions of numerical problems (NCTM, 1989).

Lots of definitions have been made by researchers on number sense in the following years. It is defined by Howden (1989) as the intuition and natural understanding of students because it has the ability to realize more than one way instead of applying the rules to reach the solution by number sense. Greeno (1991) expressed the number sense as a knowledge which is gained as a result of successful interaction with the environment; and as an ability of judging about how to find and use environmental opportunities in activities, understanding patterns, making flexible calculations by mind, estimating and reasoning about numerical values. According to Greeno's statement, the number sense is the information acquired through interaction, and it is a skill and cognitive process in which the information is used in numerical situations. The number sense, which is a wealth of mathematical knowledge, is the ability and tendency to develop effective strategies and to make mathematical reasoning in order to overcome situations that require processing (Marshall 1989). When the definitions are evaluated in general, it is seen that number sense is a cognitive process that supports making logical inference about problem situations, developing strategies, making mathematical reasoning, making decisions and noticing patterns between situations or concepts.

After creating the definitions for the number sense, the relationship between number sense and mathematics was determined and the need to observe the process of number sense was felt. Harç (2010) refers to the auxilary indicators used to define the presence or absence of the number sense ability as the number sense components. As a psychological perspective was proposed by some researchers for the number sense, a theoritical framework was formed by some researchers and some of the characteristics of number sense were defined by some

researchers. Therefore, different classifications have emerged in the literature instead of a common classification for number sense components. (Şengül and Gülbağcı Dede,2013). While creating the theoretical framework of the number sense, the number sense components are defined as number sense in the studies, the properties that are loaded on the number sense or the skills that the students have.

In the conceptual framework which they developed for the number sense exemplified the number sense with the components and the related sub-components (McIntosh, Reys and Reys; 1992). This study is the most comprehensive study in the classification of number sense components. In this study, the number sense was divided into three groups, then divided into each group of components and sub-components:

1- **Knowledge of and facility with numbers:** Feeling the regular structure of numbers, multiple representations of numbers, relative and absolute magnitudes of numbers. The references of measurement were divided into components and each components was subdivided into detailed analysis.
2- **Knowledge of facility with operations:** Understanding the effects of operations and the mathematical properties, understanding the relationship between operations are divided into components and these components are divided into sub-components.
3- **Applying knowledge of and facility with numbers and operations to computational settings:** Understanding the relationship between the problem situation, being aware of the existence of many strategies; the tendency to use a different method or presentation was examined under this group.

Another study was done by Resnick. In 1989 in the conference which was held in San Diego in order to determine the issues about the number sense and the basis of number sense, Resnick explained the possible indicators of number sense. According to him, possible indicators of number sense are as follows:

1- Using well-known number relationships to identify unsure facts.
2- Judging whether a number will be an appropriate result of a problem situation
3- Approaching the numerical answer more than calculating the exact result
4- Using the decimal structure of the number system to compose and decompose numbers to simplify operations (especially in mental calculations)
5- To be willing to make sense of situations involving numbers and quantities. Talking about numbers and the relationship of them.
6- Having an intuition about the relative magnitude of the numbers and quantities.
7- Flexible using of possible different representations of a quantity.

According to Greeno (1991) there is a need for a theoretical analysis describing the important characteristics of multiple sensation examples from a definition of the number sense term. For him, there is a need to explain how the features interact to develop this skill. Here are some of the important features of the number sense divided into three by Greeno:

1- **Flexibility in numerical calculation:** The prerequisite and component of number sense requires equivalent knowledge to be regrouped when multiplying the mind.
2- **Numerical estimating:** The ability to find the nearest numerical values when making calculations.
3- **Quantitative reasoning and inference:** This component includes inferring and reasoning about numerical values and quantities.

In his doctoral thesis, Yang (1995) described number sense components as characteristic of number sense. In this study six characteristics of number sense were determined. These are: *"well-understood number meaning, decomposition / recomposition of numbers, magnitude of number, the use of benchmarks, the relative effect of operations on numbers, flexibility with numbers and operations to computational situtations."*

Reys, Reys, McIntosh, Emanuelsson, Johansson, and Yang (1999) in their study, which measured the number sense qualifications of students aged eight to fourteen, determined six number sense components by utilizing the conceptual framework developed by McIntosh and others (1992). The following six components and the question examples of these components are: *"understanding of the meaning and size of numbers; understanding and use of equivalent representations of numbers; understanding the meaning and effect of operations; understanding and use of equivalent expressions; Flexible computing and counting strategies for mental computation, written computation, and calculator use; measurement benchmarks".*

The number sense that allows to explore patterns is related to relational understanding and meaningful learning (Berch, 2005). Because number sense is a way of thinking that supports reasoning and provides solutions to solve problems in an effective and flexible way. It supports the conceptual understanding by creating meaningful learning process in this way (Dunphy, 2007; Yang and Li, 2008; akt: Yang and Li, 2013). As a matter of fact, it is expressed that as a result of various activities and the effective teaching methods support the mathematical thoughts and learning of the students along with number sense (Markovits and Sowder, 1994; Yang at al, 2004; akt: Yang and Hsu, 2009); the lack of number sense, causes the idea that mathematics is difficult (Shumway, 2011; akt: Major and Perger, 2014).

Metacognition

Metacognition was first expressed by Flavell in 1971. Flavell used metacognition to express the control of learning and memory processes of individual (Georghiades, 2004; Hacker, 1998: akt Karakelle & Saraç, 2007). According to Flavell, metacognition is to monitor self-comprehension and self-control skills, provided that their own cognitive processes are aware of the difference and the ability to control (Özsoy, 2006). As an example for the usage of metacognition, it is necessary for an individual to realize to control C before accepting that C is the right one (Flavell, 1976; akt: Çakıroğlu, 2007). Metacognition; is a concept used to monitor and regulate cognitive processes such as learning, problem solving, comprehension, reasoning (Metcalfe and Shimamura, 1996; akt: Karakelle & Saraç, 2007, Esmer & Yorulmaz, 2017). Because metacognition; more flexible using of problem solving strategies, dealing with more difficult problems it is an extra ordinary thinking ability which helps

individuals to determine which strategy is more appropriate to use (Metcalfe, 1996; Ormrod, 2003; akt: Kaplan & Duran, 2016). Metacognition is handled under two headings: metacognitive knowledge and metacognitive control or organizing (Özsoy, 2008).

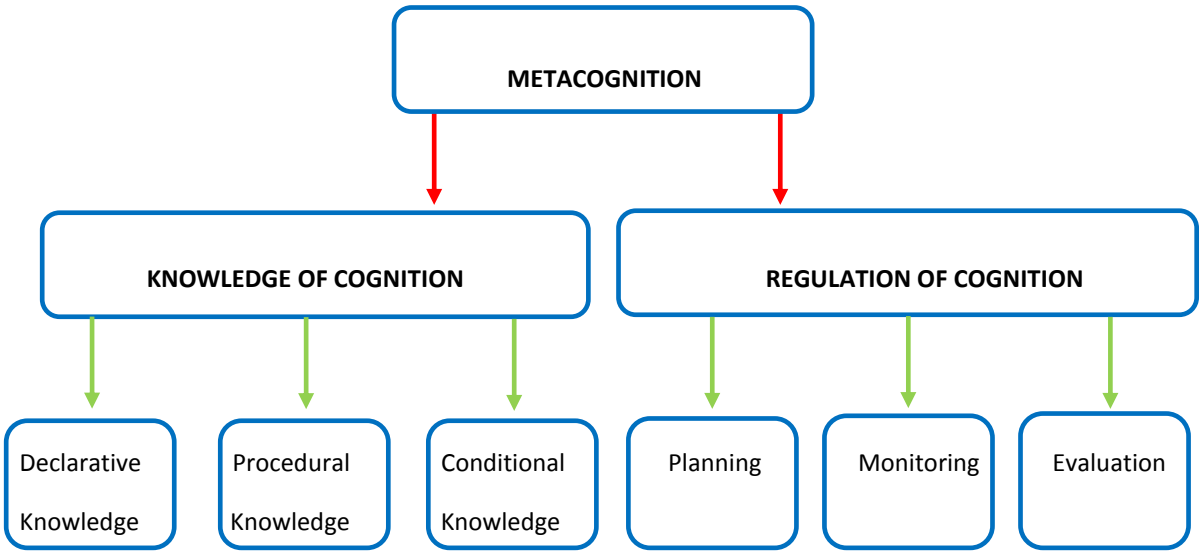

Figure 1. Metacognitive Awareness Model (Schraw ve Moshman, 1995).

The metacognition information is related to knowing for what purpose when and how to use the strategies to be used in problem solving (Flavell, 1979; akt: Aşık & Sevimli, 2015). An example is that the individual who is aware of what strategies he / she can use to solve a mathematical problem, or he / she can or cannot solve it (Özsoy, 2007). Regulation knowledge is the ability of individuals to adapt a phenomenon to their own learning through organizing information, monitoring thinking processes, controlling cognition and thinking processes, and monitoring comprehension. Regulation knowledge allows information to be used flexibly (Taras, 2005; akt: Aşık & Sevimli, 2015; Özsoy, 2006).

Because of individuals' evaluating their own thinking processes, metacognition is considered to be a high-level thinking skill (Woolfolk, 2005; akt: Kacar & Sarıçam, 2015). In this respect, metacognition is important in learning by understanding and internalizing what was learned in mathematics which is an abstract structure. Because; students with metacognition can access the information themselves through high-level thinking and questioning (Öztürk, Kurtuluş, & Aytaç, 2017). Metacognition, which helps students realize that mathematics is logical reasoning in solving mathematical problems, is also the ability to judge the solution that was obtained (Huang, A Ricci, & Mnatsakanian, 2016; Kaiser, 2007; akt: Bukova Güzel, 2016). In cases where cognitive activities are performed without a critical perspective, metacognition skills cannot be considered. For example, it is the process of passive learning in which instructions were not read, the situation was not examined, the information was copied or the formula was applied exactly without understanding. In such a learning process, students successfully mobilize their cognitive functions without any critical thinking (Georghiades, 2004; akt: Alan, 2017). Although cognitive skills were used in the given case, the metacognitive skills were not used by the students.

Applying formulas exactly, not developing strategies, not displaying critical point of view shows that the metacognitive skills are not used.

In the first phase of mathematical problem solving, metacognitive skills in terms of intuition were effective; in the last stage where the calculation results should be checked, metacognition is important in terms of evaluation (Verschaffel, 1999).

Mathematically; for being out of standard routine strategies and solutions that are very important and for being aware of what has been done in everyday life or in mathematical activities, development of metacognitive skills are required (Alan, 2017).

Relationship Between Number Sense and Metacognition[1]

As stated by Özsoy (2008) from various sources (Hacker ve Dunlosky, 2003; Huitt, 1997; Reeve ve Brown, 1985) metacognition, which is defined as the ability to control and direct its own cognitive processes and being aware of the mental activities involved in human perception, remembering and thinking is a high level managerial process in which planning, monitoring and evaluation skills are used. Metacognition, which alone does not have the ability to achieve success, serves only to learning. Metacognition is a product that occurs by the interaction of cognitive activities of individuals, their metacognitive knowledge, metacognitive experience, learning style and learning strategies (Baykara, 2011).

Carroll (1996) states that a good mental calculating that contributes to the development of metacognitive skills and the ability to predict are the evidence of the existence of number sense and he also indicates that these abilities additionally support the development of number sense. When it is considered as mental calculating and guessing is a problem-solving approach, students tend to develop their own strategies. Thus, mental calculating and guessing include high-level thinking skills (Tsao, 2004). It is similar to developing a strategy in the problem solving process by regulating one's cognitive activities in metacognition. The person who develops a strategy appropriate to the problem, who does not adhere to the rule, reviews his / her cognitive activities and exhibits high level thinking skills. According to Resnick (1989); although the number sense depends on a complex interaction between the person's knowledge, skills, the structure of the problem, the expected to solve the problem and the performance for the solution of the problem; it includes a reasoning and interpretationability that contributes to flexible thinking with numbers (Pilmer, 2008).

Metacognition which is a piece of cognition, including the ability to estimate, plan, monitor and evaluate the individual's own mental activities; does not have a place in the brain of the individual but controls many functions such as perception and attention (Baykara, 2011; Brown, 1980, akt: Özsoy, 2008). While Resnick (1989) describes the number sense as a concept related to higher-order thinking skills; Carpenter (1989) argues that the sense of

[1] This section was written using the doctoral thesis which name is " *4. Sınıf Öğrencileri için Sayı Hissi Testi'nin Geliştirilerek Öğrencilerin Sayı Hislerinin İncelenmesi"*, of the first author.

number is not a collection of information but a way of thinking. When the definitions are considered together, for the good usage of number sense which is a way of thinking, skills such as perception, attention, flexible thinking, strategy development are required. Since metacognition is a part of the cognition that controls these skills, it also has the task of controlling the number sense. It also shows that the abilities of a person in the metacognition concept to make predictions about his / her own cognitive activities, to observe and evaluate the process, to determine new strategies appropriate to the situation show us that he / she reconsiders and finds and uses the new ways out appropriate to the situation. Similar processes are observed in number sense, too. This relationship resembles to the links between the numbers in number sense and operations and it is possible to estimate the result of the problem state in number sense, to judge the result, to make inference, to develop effective and appropriate new strategies. These properties of metacognition and number sense, indicate that there is a relationship between two concepts.

Anghileri (2006) states that the situations in which students who have number sense create links in problem solving activities and express answers are the examples that developing number sense requires metacognition. The effectiveness of thinking strategies used by students may vary depending on the versatility in the way students understand. The sense of number thus creates multiple methods and solutions (McIntosh et al., 1997; akt: Pilmer, 2008). Based on all these explanations, the relationship between metacognition and number sense can be modeled as follows:

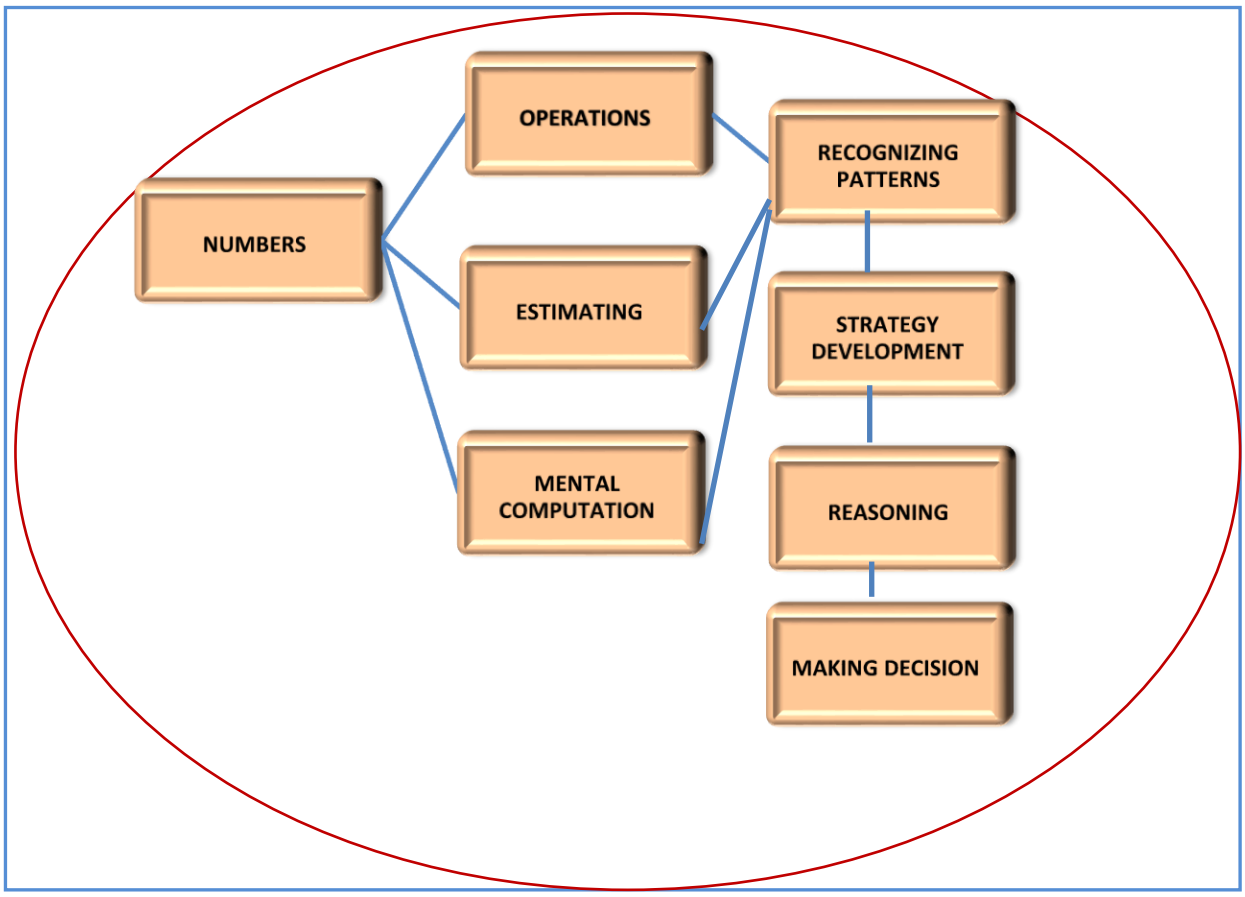

Figure 2. Component Analysis Number Sense and Its Relationship with Metacognition

McIntosh, Reys and Reys (1992) say that the links between numbers and operations are similar to those used in metacognition and problem solving activities. According to the researchers, the individual who has a good number sense; thinks about numbers and operations and gets the result that needs to be achieved. While applying the number sense, the individual plans the problem, monitors the process and evaluates the result. While doing all this, he is able to control and guide his own cognitive processes in the awareness of the mental activities involved in perception, recall and thinking. This situation is in parallel with the process of using number sense and metacognitive process.

A good understanding a mathematical concept depends on the correct, precise and numerous connections between all aspects of that concept (Hiebert ve Carpenter, 1992; akt: Tsao, 2004). The degree of understanding the conceptual information of a student is determined by the number of connections between concepts, accuracy and precision between links (Tsao, 2004). According to Ekenstam (1977), number sense involves the development of different relationships between mathematical concepts, knowledge and skills. Thus, many concepts are reached at the same time when needed (Şengül, 2013). Students who use number sense at a good level; can relate mathematical numbers and numerical expressions to real-world quantities. Students will not have problems in this way; they will be able to visualize numbers with multiple representations, see number patterns and notice numerical errors (Case, 1998; akt: Menon, 2004). In short, the number sense focuses on conceptual understanding (Yang, 2003). The number sense that provides the development of conceptual understanding, involves relational understanding, thus meaningful learning.

CONCLUSION and DISCUSSION

In today's conditions where science and technology have gained speed and individuals compete in order to reach them and produce more; it is unavoidable to have high level thinking skills such as practicality on mathematical operations, reconsidering, reasoning, mentally calculating, problem solving, estimating, thinking flexibly with thinking skills and strategies. It has been ensured that the number sense that generally refers to the desired mathematical skills has enabled students to use and develop a cognitive skills actively by influencing mathematics achievement positively (Maryam, Mahnaz ve Hasan, 2011; Mohamed ve Johnny, 2010; Yang, Li ve Li, 2008), supporting mathematical thinking and learning (Markovits ve Sowder, 1994; Yang et al., 2004; akt: Yang ve Hsu, 2009); increasing the math achievement of metacognition and the success in problem solving (Md. Yunus & Ali, 2008; Schneider & Artelt, 2010; Shraw & Dennison, 1994). The number sense, such as metacognition which is a kind of reasoning, is also necessary for mathematical thoughts and reasoning (Kaiser, 2007; akt: Bukova Güzel, 2016; Shumway, 2011; akt: Major ve Perger, 2014). These findings indicate the existence of a relationship between number sense and metacognition.

Students who cannot use the number sense fully and do not understand the relationships between concepts have to learn and remember many rules in order to overcome the problems they face in their daily life. At the same time, this situation shows that it is not able to adequately display the metacognition skills, which require skills in strategy development and reasoning problems. However, the results of adult studies reveal that about

80% of the mathematical calculations in daily life require mental regulation rather than rule based calculations (Reys ve Reys, 1995; akt: Pilmer, 2008). Similarly, although students can learn the necessary formulas and apply them to textbooks and test cases, they cannot answer the question as they do not understand the conceptual relations when they encounter new problems (Von Glaserfeld, 1995; akt: Yong & Kiong, 2005). The results of the research suggest that number sense and metacognitive skills are the skills that should be acquired by the individual as much as needed in daily life.

Using of knowledge by adapting to different situations, connecting between all aspects of a concept, visualizing the numerical expressions in multi ways, noticing the numerical errors and to be able to make connection between the numerical expressions and daily life require thinking flexibly. As number sense is a skill based on flexible thinking (Yang ve Hsu, 2009), it can be seen both as a prerequisite and component of number sense, depending on flexible thinking and meaningful learning. At the same time it is a thinking skill required for more flexible use of problem solving strategies in metacognition (Metcalfe, 1996; Ormrod, 2003; akt: Kaplan & Duran, 2016).

SUGGESTIONS

In this study, number sense and metacognition were emphasized and the relationship between them was examined. According to the findings obtained from the literature, it is possible to make studies on the relationship between number sense and metacognition in different levels of education with students. The researchs can be done using both quantitative and qualitative methods. This study is thought to enlighten on further research. As it is understood from the results, number sense and metacognition are significant in mathematics. Taking this into consideration, the Ministry of National Education may develop activity for mathematic lessons in an effective way.

SAYI HİSSİ VE ÜSTBİLİŞ İLİŞKİSİ

TÜRKÇE GENİŞ ÖZET

GİRİŞ

Gelişmekte olan günümüz koşullarında bilginin önemi, bilgiye ulaşma yolları, öğrenme-öğretme yaklaşımları, teknoloji gibi birçok alanda değişim yaşanmaktadır. Özellikle bilim ve teknolojide yaşanan gelişmeler, bilgiyi pasif olarak edinmek yerine bilgiye bireysel çabalar yoluyla ulaşılmasını geçerli kılmaktadır. Bilgiye ulaşabilmek için bireylerin problem çözme, kendi öğrenmelerini düzenleyebilme, edindiği bilgileri yeni durumlara transfer etme, muhakeme etme, akıl yürütme, eleştirel düşünme, yaratıcı ve özgün stratejiler geliştirme gibi becerilere sahip olmaları ve bu becerileri sosyal hayatlarında kullanabilmeleri gerekmektedir. Bu becerilerin kazanılmasında ve günlük hayat durumlarında kullanılabilmesindematematik büyük bir öneme sahiptir.

Hayat boyu ihtiyaç duyulacak matematiğin belirli amaçlar etrafında ele alınması okul yıllarında olmaktadır. Okul matematiği ile bireylere kazandırılması istenilen beceriler, öğretim programları dâhilinde yapılmaktadır. 2005 yılında çağın gereklerine göre güncellenerek uygulamaya konulan matematik dersi öğretim programı ilekavramları ilişkilendirme, akıl yürütme, esnek düşünme, problem çözme, tahminde bulunma ve zihinden işlem yapmagibi matematiksel becerileri kazandırmak amaçlanmıştır. Açıkça ifade edilmeyen fakat öğrencilere kazandırılması amaçlanan beceriler yoluyla bileşensel analizi verilen sayı hissi kavramına programda yer verilmiştir (MEB, 2005). Süreç içerisinde program güncellemeleri devam etmiş, son güncelleme ise 2018 yılında yapılmıştır. 2018 yılında güncellenen matematik dersi öğretim programında öğrencilerin üstbilişsel becerileri kullanabilmeleri amacıyla konular ile ön öğrenmeler arasında ilişki kurulmuş; değerler, beceriler ve yetkinlikler kapsamında diğer disiplin alanları ve günlük hayatla bütünleştirilmiş; bu yolla üstbilişsel bilgi ve becerilerin geliştirilmesine öncekilere göre daha fazla önem verilmiştir (MEB, 2018).

Okul matematiğindeki önemli kavramlar olarak karşımıza çıkan; 21. yüzyılda matematik eğitimi alanında anahtar bir içeriğe sahip olacağı belirtilen; matematik eğitiminin merkezine konulan ve okul matematiğinin temel amacı olarak vurgulanan sayı hissi (Yang ve Li, 2008; NCTM, 2000; akt: Yang, 2003) ile üst düzey düşünme becerileri olarak ifade edilen üstbiliş (Woolfolk, 2005; akt: Kacar & Sarıçam, 2015) matematik dersi öğretim programında yerini almıştır.

Matematik dersi akademik başarısıile gerek sayı hissi (Çekirdekci, Şengül & Doğan, 2016; Harç, 2010; Jordan, Glutting ve Ramineni, 2009) gerekse üstbilişarasında anlamlı ilişki bulunduğu (Aşık & Sevimli, 2015; Memiş & Arıcan, 2013) göz önüne alındığında, bu iki kavram arasındaki ilişki durumunun araştırılmasının gerekli olduğu düşünülmektedir.

Çalışmanın amacı matematik eğitiminde özellikle son yıllarda sıklıkla vurgusu yapılan üstbiliş ve sayı hissi kavramlarına ait tanımları, matematik eğitimindeki önemlerini açıklamak ve sayı hissi ile üstbiliş arasındaki ilişkiyi incelemektir.

YÖNTEM

Çalışmada sayı hissi ve üstbiliş literatür çerçevesinde ele alınarak aralarındaki ilişki açıklanmıştır. Bu nedenle bu çalışma, verilerin sentezlenmesine dayanan bir literatür taraması şeklinde tasarlanmıştır. Kuramsal bir nitelik taşıyan bu çalışmada, ilgili literatür kapsamında mevcut durum betimlenerek analiz edilmiştir. Bu nedenle çalışmada nitel veri toplama yöntemlerinden doküman incelemesine başvurulmuştur. Doküman incelemesi araştırmanın hedeflerine yönelik bilgi içeren yazılı materyallerin analizini kapsar (Yıldırım ve Şimsek, 2006). Çalışmada ilgili literatür taranmış, ulaşılan araştırmalar incelenmiş ve elde edilen bilgiler tartışılarak sayı hissi ile üstbiliş arasındaki ilişki ele alınmıştır.

TARTIŞMA VE SONUÇ

Bilim ve teknolojinin hız kazandığı, bireylerin bunlara yetişebilmek, daha fazla üretim yapabilmek adına zamanla yarıştığı günümüz koşullarında, işlem becerilerinde pratiklik, akıl yürütme, muhakeme, zihinden işlem yapma, problem çözme, tahminde bulunma, esnek düşünme ile düşünme becerileri ve stratejileri gibi üst düzey düşünme becerilerine sahip olmak kaçınılmazdır. Kazandırılması istenilen matematiksel becerileri genel olarak ifade eden sayı hissinin matematik başarısını olumlu yönde etkilemek (Maryam, Mahnaz ve Hasan, 2011; Mohamed ve Johnny, 2010; Yang, Li ve Li, 2008), matematiksel düşünceyi ve öğrenmeyi de desteklemek (Markovits ve Sowder, 1994; Yang et al., 2004; akt: Yang ve Hsu, 2009); üstdüzey düşünme becerisi olan üstbilişin matematik başarısını artırmak, problem çözme başarısını yükseltmek yoluyla (Shraw&Dennison, 1994; Md. Yunus & Ali, 2008; Schneider & Artelt, 2010) öğrencilerin bilişsel becerileri aktif olarak kullanmaları ve geliştirmeleri sağlanmıştır. Bir tür muhakeme etme yeteneği olan üstbiliş (Kaiser, 2007; akt: Bukova Güzel, 2016) gibi sayı hissi de matematiksel düşünceler ve muhakeme yeteneği için oldukça gereklidir (Shumway, 2011; akt: Major ve Perger, 2014). Literatür kapsamında ulaşılan bu bulgular matematik üzerinde etkiye sahip olan sayı hissi ve üstbiliş arasında bir ilişkinin varlığını göstermektedir.

Sayı hissini tam olarak kullanamayan ve kavramlar arasındaki ilişkileri kavramayan öğrenciler günlük hayatta karşılaştıkları sorunların üstesinden gelebilmek için birçok kuralı öğrenmek ve hatırlamak zorunda kalmaktadır. Aynı zamanda bu durum strateji geliştirme ve problemleri muhakeme etme becerileri gerektiren üstbiliş becerisini yeter düzeyde sergileyemediğini göstermektedir. Ancak yetişkinler üzerinde yapılan araştırmaların sonuçları günlük hayattaki matematiksel hesaplamaların yaklaşık %80'inin kural temelli hesaplamalar yerine zihinsel düzenlemelere ihtiyaç duyulduğunu ortaya koymaktadır (Reys ve Reys, 1995; akt: Pilmer, 2008). Benzer şekilde öğrenciler, gerekli formülleri öğrenerek bunları ders kitapları ve test durumlarına uygulayabilmelerine rağmen; yeni problemler ile karşılaştıklarında kavramsal ilişkileri anlamadıkları için soruyu cevaplandıramamaktadır (VonGlaserfeld, 1995; akt: Yong & Kiong, 2005). Çok sayıda araştırmanın sonuçları da

(Bell, 1974; Markovits ve Sowder, 1994; McIntosh, Reys ve Reys, 1997; Singh, 2009; Yang ve Hsu 2009) sayı hissi ve üstbiliş becerilerinin günlük hayatta ihtiyaç duyulduğu kadar birey tarafından kazanılması gereken beceriler olduğu sonucuna götürmektedir.

Bilginin farklı durumlara uyarlanarak kullanılması, bir kavramın tüm yönleri ile arasında bağlantı kurulması, sayısal ifadelerin çoklu yollarla görselleştirilmesi, sayısal hataların farkedilmesi ve matematikteki sayısal ifadeler ile günlük hayat arasında bağlantı kurulması esnek düşünmeyi gerektirir. Sayı hissi esnek düşünmeye dayalı bir beceri olduğundan (Yang ve Hsu, 2009), esnek düşünme anlamlı öğrenmeye bağlı olarak sayı hissinin hem önkoşulu hem de bileşeni olarak görülebilir. Aynı zamanda üstbilişte problem çözme stratejilerinin daha esnek kullanılması için gereken bir düşünme becerisidir (Metcalfe, 1996; Ormrod, 2003; akt: Kaplan & Duran, 2016).

Bu çalışmada, sayı hissi ve üstbiliş üzerinde durulmuş, sayı hissi ve üstbiliş arasındaki ilişki incelenmiştir. Literatüre dayalı olarak elde edilen bulgulara göre sayı hissi ile üstbiliş arasındaki ilişkinin farklı eğitim kademelerindeki öğrencilerin katılacağı çalışmalar yapılabilir. Hem nicel hem de nitel yöntemlerin kullanıldığı araştırmalar yapılabilir. Bu yönüyle yapılan çalışmanın sonraki araştırmalara ışıktutacağı düşünülmektedir. Elde edilen sonuçlardan anlaşıldığı gibi, matematikte sayı ve metabiliş önemlidir. Bunu dikkate alarak, Milli Eğitim Bakanlığı matematik dersleri için etkin bir şekilde etkinlik geliştirilebilir.

Anahtar Kelimeler: Matematik, sayı hissi, üstbiliş.

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